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Instantons of M(atrix) theory in PP-wave background

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Abstract: M(atrix) theory in PP-wave background possesses a discrete set of classical vacua, all of which preserves 16 supersymmetry and interpretable as collection of giant gravitons. We find euclidean instanton solutions that interpolate between them, and analyze their properties. Supersymmetry prevents direct mixing between different vacua but still allows effect of instanton to show up in higher order effective interactions, such as analog of v^4 interaction of flat space M(atrix) theory. An explicit construction of zero modes is performed, and Goldstone zero modes, bosonic and fermionic, are identified. We further generalize this to massive M(atrix) theory that includes fundamental hypermultiplets, corresponding to insertion of longitudinal fivebranes in the background. After a brief comparison to their counterpart in $AdS \times S$, we close with a summary.

Keywords: M(atrix) Theories, Penrose limit and pp-wave background, Solitons Monopoles and Instantons.

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1. Introduction

Maximally supersymmetric plane waves background in eleven dimensions is obtained from either $AdS_7 \times S^4$ or $AdS_4 \times S^7$ by taking the Penrose limit [1]. String theory or M-theory in the plane waves background [2] admits rich tractable structures which are useful to understand AdS/CFT duality and physics in the presence of Ramond-Ramond flux: Lightcone Green-Schwarz action of type-IIB string theory is free and exactly quantizable [3]. M-theory in eleven dimension has a M(atrix) theory formulation with a mass parameter μ which can be used as an expansion parameter in appropriate combination with usual Yang-Mills coupling [4, 5].

For M(atrix) theory in the plane waves background [2], first written down by Berenstein, Maldacena, and Nastase (BMN), the vacua are characterized by discrete set of fuzzy spheres which preserve all 16 dynamical supersymmetries. They are nothing but D0-branes expanded by Myers' dielectric term [6]. Tracing back to $AdS \times S$ space, these fuzzy spheres originate from giant gravitons rotating with the velocity of light in the AdS space or in the sphere [7, 8, 9]. One purpose of this note is to examine physics of giant gravitons in this more tractable quantum theory.

Given such degenerate set of vacua, an obvious thing to ask is whether there are instantons that interpolate between different classical vacua. It turns out that search for instantons is greatly simplified thanks to BPS-like properties of the latter. That is, instantons interpolating between fully supersymmetric vacua of this theory preserves half of $\mathcal{N} = 16$ dynamical supersymmetry. Those instantons that interpolate between any nontrivial vacuum and the perturbative vacuum are particularly simple, it turns out, and an explicit counting of zero modes, bosonic and fermionic, is carried out. As supersymmetry may already suggests, these instantons have too many fermionic Goldstone zero modes to mix vacua, and will contribute to higher order interaction terms only.

This note is organized as follows. After a brief review of the BMN M(atrix) theory in section 2, we take analytic continuation into the euclidean time in section 3. Here we have to deal with subtlety associated with Majorana condition on fermions, utilizing "double fermion technique" [10]. BPS-like equation follows then, either by writing euclidean action in a complete square form or by writing down supersymmetry condition. We solve for all instantons that interpolate between the perturbative vacuum and arbitrary collections of giant gravitons.

In section 4, we construct bosonic and fermionic zero modes around these instantons explicitly. Apart from 8 fermionic zero modes and a single bosonic zero mode, each arising from spontaneously broken symmetry, we find a large number of extra zero modes for most of these instantons. This leads us to believe that there are more diverse form of instantons, which we confirm in part in section 5.

Section 6 repeats the computation for certain mass-deformed $\mathcal{N} = 8$ SYQM [11]. The latter have been discovered and identified as M(atrix) theory in the presence of longitudinal fivebranes . Among the vacua of this latter theory are giant gravitons trapped by the longitudinal fivebranes. We again isolate and study supersymmetric instantons that interpolate from a generic vacuum to the perturbative one. After a review of giant gravitons and instantons in $AdS \times S$, the Penrose limit thereof in section 7, we close with summary.

2. BMN Matrix theory and classical vacua

We start with a summary of the mass deformed BMN M(atrix) theory [2]. Here we review some basic properties of the theory, in part to set up the notation, and recall its classical vacuum structure.

2.1 Lagrangian and supersymmetry

Introducing 16×16 real and symmetric SO(9) Dirac matrices,

$$\gamma_I \qquad (I = 1, 2 \ldots 9) \tag{2.1}$$

the lagrangian of the mass-deformed M(atrix) theory of BMN is

$$\begin{aligned} L = &\frac{1}{2}\mathrm{Tr}\left(\sum_I (D_0 X_I)^2 + \frac{1}{2}\sum_{IJ}[X_I, X_J]^2 + i\Psi^T D_0\Psi - \Psi^T\gamma_I[X_I, \Psi]\right) + \\ &+ \frac{1}{2}\mathrm{Tr}\left(-\left(\frac{\mu}{3}\right)^2 \sum_{a=5,6,7}(X_a)^2 - \left(\frac{\mu}{6}\right)^2 \sum_{s=1,2,3,4,8.9}(X_s)^2\right) + \\ &+ \frac{1}{2}\mathrm{Tr}\left(\frac{2i\mu}{3}\epsilon_{abc}X_a X_b X_c - \frac{i\mu}{4}\Psi^T\gamma_{567}\Psi\right), \end{aligned} \tag{2.2}$$

Here the fermion Ψ is Majorana.[1] In addition, both X^I and Ψ are valued in $N \times N$ hermitian matrices for an integer $N \geq 1$. Dimensionful quantity μ introduces a mass-gap to the system and make the spectrum of the theory completely discrete. For the most part, we will be concerned with ground state sector of this theory.

This lagrangian was shown to be invariant under the following 16 (dynamical) supersymmetry transformation

$$\begin{aligned} \delta A_0 &= i\Psi^T\epsilon, \\ \delta X_I &= i\Psi^T\gamma_I\epsilon, \\ \delta\Psi &= \left(D_0 X_I\gamma_I - \frac{i}{2}[X_I, X_J]\gamma_{IJ}\right)\epsilon + \left(\frac{\mu}{3}\sum_{a=5,6,7} X_a\gamma_a\gamma_{567} - \frac{\mu}{6}\sum_{s=1,2,3,4,8,9} X_s\gamma_s\gamma_{567}\right)\epsilon. \end{aligned} \tag{2.5}$$

provided that we force the following explicit time-dependence of the transformation parameter ϵ,

$$\epsilon(t) = e^{-\frac{\mu}{12}\gamma_{567}t}\epsilon_0, \tag{2.6}$$

which makes the superalgebra quite unconventional. In particular, supercharges do not commute with the hamiltonian, but raise or lower energy by $\mu/12$ unit. The notion of "supermultiplet" no longer implies degeneracy in this deformed superalgebra.

[1] The Majorana condition in 9+1 dimensions is achieved by imposing a Majorana condition,

$$\Psi = C\bar{\Psi}^T \tag{2.3}$$

for some charge conjugation matrix C. For 16-component fermion here, we may introduce B which is a 16×16 block off $C(\Gamma^0)^T$ such that

$$\Psi = B\Psi^*, \qquad B(\gamma^I)^* B^{-1} = \gamma^I, \qquad BB^* = 1 \tag{2.4}$$

In the present case, we may choose $B = 1$ so that γ^I's are real and symmetric.

2.2 Classical vacua are collections of giant gravitons

The potential of the mass-deformed matrix theory of BMN lifts flat directions altogether, and instead leave behind a set of discrete vacua. One way to see this is to realize that the potential can be written in a complete square form. Setting $X^i = 0$ for $i = 1, 2, 3, 4, 8, 9$ as a matter of convenience, the potential reduces to

$$V \to \mathrm{Tr}\left(-\frac{1}{4}\sum_{a,b=5,6,7}[X^a, X^b]^2 + \frac{1}{2}\left(\frac{\mu}{3}\right)^2\sum_{a=5,6,7}(X^a)^2 - i\frac{\mu}{3}\epsilon_{abc}X^aX^bX^c\right), \qquad (2.7)$$

which can be made into a complete square,

$$-\frac{1}{4}\sum \mathrm{Tr}\left(\left[X^b, X^c\right] + \frac{i\mu}{3}\epsilon_{bca}X^a\right)^2. \qquad (2.8)$$

Thus zero of the potential is given by X^a's satisfying a set of commutation relationship

$$\left[X^a, X^b\right] = -\frac{i\mu}{3}\epsilon^{abc}X^c\,, \qquad (2.9)$$

and classical vacua are given by static configurations of the form

$$X^a = -\frac{\mu}{3}J^a\,, \qquad (2.10)$$

with any N dimensional representation J^a of SU(2).

Nothing here requires J's to form an irreducible representation, so we should look for general N dimensional representation. We may do this by filling out three $N \times N$ matrices with various irreducible representations of SU(2). Different ordering among such irreducible blocks are all gauge equivalent, and therefore classification of vacua comes down to classifying inequivalent partitions of the integer N.

These classical vacua are reminiscent of bound states of original M(atrix) theory [12]. In the latter, the bound states are supposed to exist for any number of D0's, which means that Hilbert space contain sectors of n_p particles bound together for any partition of N ($N = \sum_p n_p$). Given the U(N) BMN M(atrix) theory, there is precisely one such distinct state for each partition $N = \sum_p n_p$, corresponding to a direct sum of n_p dimensional irreducible representations of SU(2). These classical states (and their quantum counterparts) are closest thing we have to usual Kaluza-Klein modes of supergraviton, or equivalently BPS bound states of D-particles. In fact, one may think each of these $n_p \times n_p$ blocks as the bound state n_p D-particles which are blown-up into a spherical membrane due to Myers' dielectric effect [6]. In the mass deformed version here, flat directions are lost and everything is confined near the origin, yet it is tantalizing that hint of these D0 bound states still survives in the form of discrete set of classical vacua [7].

3. Euclidean instantons

3.1 Euclidean action and supersymmetry

We wish to look for interpolating solution between different vacua, so a natural thing to do is to euclideanize the theory. The upshot is that we can do this effectively by mapping

$$t \to -i\tau \qquad (3.1)$$

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which is actually is a result of more careful operation that maintains 10-dimensional origin for fermions. Rest of this subsection will address how to do this more carefully.

Analytic continuation to the euclidean signature is often hazardous when real fermions are present. The problem with real fermions in field theory is that representation theory under $SO(d,1)$ Clifford algebra is different from that under $SO(d+1)$. Since fermions of this quantum mechanics came from $SO(9,1)$ spinors, this sort of problem again shows up when we try to perform a euclidean continuation of this quantum mechanics. On the other hand, this also tells us that the trick we can use is already available in field theory language, where we abandon hermiticity of lagrangian, "double" fermion content by regarding the conjugated fermions as independent, and then impose certain "Majorana-like" constraints on the pair [10].

Let us start by rewriting the Minkowski action in $SO(9,1)$ spinor notation.

$$\begin{aligned}
L_M = & \frac{1}{2}\mathrm{Tr}\left(\sum_I (D_0 X_I)^2 + \frac{1}{2}\sum_{IJ}[X_I,X_J]^2 - i\bar{\Psi}\Gamma^0 D_0\Psi - \bar{\Psi}\Gamma_I[X_I,\Psi]\right) + \\
& + \frac{1}{2}\mathrm{Tr}\left(-\left(\frac{\mu}{3}\right)^2 \sum_{a=5,6,7}(X_a)^2 - \left(\frac{\mu}{6}\right)^2 \sum_{s=1,2,3,4,8.9}(X_s)^2\right) + \\
& + \frac{1}{2}\mathrm{Tr}\left(\frac{2i\mu}{3}\epsilon_{abc}X_aX_bX_c - \frac{i\mu}{4}\bar{\Psi}\Gamma_{567}\Psi\right),
\end{aligned} \tag{3.2}$$

where Ψ is Majorana-Weyl spinor and 10 dimensional Γ matrices are given

$$\begin{aligned}
\Gamma^0 &= 1\otimes i\sigma_2\,, \\
\Gamma_I &= \gamma_I \otimes \sigma_1 \qquad (I = 1,2\ldots 9)\,, \\
\Gamma^{11} &= 1\otimes\sigma_3\,,
\end{aligned} \tag{3.3}$$

where γ_I are usual 16×16 hermitian $SO(9)$ Dirac matrices.

Weyl spinor Ψ is subject to the chirality condition

$$\Gamma_{11}\Psi = -\Psi\,. \tag{3.4}$$

which effectively reduces Ψ to be 16-component, justifying the fact that the same notation Ψ is used for 16-component spinor of previous section and for the current 32-component spinor with upper half identically zero. In particular this means that 32×32 matrices Γ's are effectively 16×16 matrices of the following form,

$$\begin{aligned}
\Gamma^0 &\to +1\,, \\
\Gamma_I &\to \gamma_I\,,
\end{aligned} \tag{3.5}$$

when acting on Ψ directly. Finally, the Majorana condition on Ψ is

$$\Psi = C\bar{\Psi}^T\,, \tag{3.6}$$

where the charge conjugation matrix C is chosen to be

$$C = 1\otimes i\sigma_2\,, \tag{3.7}$$

which satisfies

$$C\Gamma_I = -\Gamma^T C_I \,, \qquad C^T = -C \,. \tag{3.8}$$

This gives the Minkowski action of the previous section.

We may euclideanize the action and the supersymmetry by taking

$$t = -i\tau \quad (\Rightarrow D_0 = iD_\tau) \,, \qquad \Gamma^0 = -i\Gamma_{10} \,, \tag{3.9}$$

where

$$\Gamma_{10} = 1 \otimes \sigma_2 \,. \tag{3.10}$$

Imposing euclidean Weyl condition on the fermions, we have effective reduction of Dirac matrices to 16×16.

$$\begin{aligned} \Gamma_{10} &\to -i \\ \Gamma_I &\to \gamma_I \\ \Gamma_{10}\Gamma_I &\to i\gamma_I \,, \end{aligned} \tag{3.11}$$

when acting on Ψ directly. Furthermore, we use "fermions doubling" trick, introducing $\bar{\chi}$ instead of $\bar{\Psi}$.

$$\begin{aligned} L_E = {} & \frac{1}{2}\mathrm{Tr}\left(\sum_I (D_\tau X_I)^2 - \frac{1}{2}\sum_{IJ}[X_I, X_J]^2 + \bar{\chi} D_\tau \Psi + \bar{\chi}\gamma_I [X_I, \Psi]\right) \\ & + \frac{1}{2}\mathrm{Tr}\left(+ \left(\frac{\mu}{3}\right)^2 \sum_{a=5,6,7} (X_a)^2 + \left(\frac{\mu}{6}\right)^2 \sum_{s=1,2,3,4,8.9} (X_s)^2 \right) \\ & + \frac{1}{2}\mathrm{Tr}\left(-\frac{2i\mu}{3}\epsilon_{abc} X_a X_b X_c + \frac{i\mu}{4}\bar{\chi}\gamma_{567}\Psi \right) . \end{aligned} \tag{3.12}$$

Without Majorana condition, the path integral measure would be

$$[d\Psi][d\bar{\chi}] \,. \tag{3.13}$$

However, we must impose a "Majorana-like" condition on the pair

$$\bar{\chi} = \Psi^T C \,, \tag{3.14}$$

where C is the same charge conjugation matrix in Minkowski signature. The measure is effectively,

$$[d\Psi] \,. \tag{3.15}$$

This is analogous to contour integral over complex bosonic coordinates.[2]

[2] Note also that we have abandoned hermiticity of the lagrangian in doing this. This is not something new in fact. The same happens even for bosons when we start with canonical variables. For fermions, in a sense, we are always working with canonical variables.

Supersymmetry transformations remain intact once Wick rotation is performed. Transformation of bosons are

$$
\begin{aligned}
\delta A_{10} &= \frac{1}{2}(\bar{\chi}\epsilon - \bar{\eta}\Psi)\,, \\
\delta X_I &= \frac{i}{2}(\bar{\chi}\gamma_I\epsilon - \bar{\eta}\gamma_I\Psi)\,,
\end{aligned}
\tag{3.16}
$$

where $\bar{\eta}$ is related to ϵ via

$$
\bar{\eta} = \epsilon^T C \to -\epsilon^T\,. \tag{3.17}
$$

Transformation of fermions are such that,

$$
\begin{aligned}
\delta\Psi &= \left(iD_\tau X_I\gamma_I - \frac{i}{2}[X_I, X_J]\gamma_{IJ}\right)\epsilon \\
&+ \left(\frac{\mu}{3}\sum_{a=5,6,7} X_a\gamma_a\gamma_{567} - \frac{\mu}{6}\sum_{s=1,2,3,4,8,9} X_s\gamma_s\gamma_{567}\right)\epsilon\,,
\end{aligned}
\tag{3.18}
$$

from which transformation of $\bar{\chi}$ may also be inferred. Time-dependence of supersymmetry parameter must be analytically continued as well,

$$
\epsilon(\tau) = e^{i\frac{\mu}{12}\gamma_{567}\tau}\epsilon_0\,. \tag{3.19}
$$

3.2 BPS bound and unbroken supersymmetry

With a discrete set of vacua, all degenerate with each other, it is natural to expect an instanton that interpolate between such vacua. Large number of supersymmetry tends to prohibit a direct mixing of classical vacua even if an instanton is present, but it does not mean that instantons do not enter physical processes. We will come back to relevancy of instanton in a later section. These issues will be addressed in section 4 in some detail, but let us gather some elementary facts about euclidean equation of motion.

Gathering bosonic part of euclidean action that involves the three bosonic quantity X^a's,

$$
S_E = \frac{1}{2}\int d\tau \sum_a \mathrm{Tr}\left\{(D_\tau X_a)^2 + \left(\frac{\mu}{3}X^a - i\epsilon^{abc}X^bX^c\right)^2\right\} + \cdots\,, \tag{3.20}
$$

we may complete the square one more time,

$$
\begin{aligned}
S_E &= \frac{1}{2}\int d\tau \sum_a \mathrm{Tr}\left(\mp D_\tau X_a + \frac{\mu}{3}X^a - i\epsilon^{abc}X^bX^c\right)^2 \pm \\
&\pm \int d\tau \sum_a \mathrm{Tr}\left((D_\tau X_a)\left(\frac{\mu}{3}X^a - i\epsilon^{abc}X^bX^c\right)\right) + \cdots\,.
\end{aligned}
\tag{3.21}
$$

The ellipsis denotes the remainder of the action, which are composed of either a nonnegative part from the rest of X^I's or fermion bilinears, none of which would be involved in the interpolating solution.

Setting the other fields to vanish, we find a topological lower bound of the action from the second line,

$$S_E \geq \pm \int d\tau \frac{d}{d\tau} \sum_a \mathrm{Tr}\left(\left(\frac{\mu}{6} X^a X^a - i\frac{1}{3}\epsilon^{abc} X^a X^b X^c\right)\right) =$$
$$= \pm \mathrm{Tr}\left(\frac{\mu}{6} X^a X^a - i\frac{1}{3}\epsilon^{abc} X^a X^b X^c\right)\Big|_{-\infty}^{\infty} . \tag{3.22}$$

For configurations interpolating between two distinct vacua, we require

$$\begin{aligned} X^a(\infty) &= -\frac{\mu}{3} J^a_+ , \\ X^a(-\infty) &= -\frac{\mu}{3} J^a_- , \end{aligned} \tag{3.23}$$

With this, the lower bound of the euclidean action collapses to a trace over the quadratic Casimir invariant,

$$S_E \geq \pm \frac{1}{6}\left(\frac{\mu}{3}\right)^3 \left(\mathrm{Tr}\, J^a_+ J^a_+ - \mathrm{Tr}\, J^a_- J^a_-\right) . \tag{3.24}$$

Thus, to find an instanton solution between a pair of different vacua, it suffices to solve the first order equations,

$$\pm D_\tau X^a = \frac{\mu}{3} X^a - i\epsilon^{abc} X^b X^c \tag{3.25}$$

under the boundary condition (3.23). The sign is chosen so that the lower bound for the euclidean action is nonnegative. It is not difficult to check that any solution to this equation also solves full equation of motion.

Any euclidean solution that solves eq. (3.25) enjoys a special property that it is preserved by half of 16 dynamical supersymmetries. Wick rotating the SUSY transformation and isolating transformation property of the fermions, we find the following terms

$$\delta\Psi = \left(iD_\tau X_a \gamma_a - \frac{i}{2}[X_a, X_b]\gamma_{ab} + \frac{\mu}{3} \sum_{a=5,6,7} X_a \gamma_a \gamma_{567} \right) \epsilon \tag{3.26}$$

on the instanton background. On the other hand, we have the identity,

$$\gamma_{ab} = \gamma_c \gamma_{567} \epsilon^{abc} , \tag{3.27}$$

so that

$$\delta\Psi = i\left(D_\tau X_a \gamma_a - \left(\frac{\mu}{3} X_a - i\epsilon^{abc} X^b X^c\right) \gamma_a (i\gamma_{567})\right) \epsilon = 0 , \tag{3.28}$$

provided that

$$i\gamma_{567}\epsilon_0 = \pm\epsilon_0 , \tag{3.29}$$

where the sign choice should be correlated with that of (3.25). With each choice of sign, then, these are the 8 preserved supersymmetries promised. For this reason, we refer to (3.25) as the BPS equation.

3.3 Instanton solutions

An infinite class of instantons can be found with a simplifying condition, $J_+^a = 0$. This gives interpolating configuration from an arbitrary vacuum at past infinite to the trivial perturbative one at future infinity. The solution is fairly elegant. We use the ansatz,

$$X^a = -\frac{\mu}{3} f(\tau) J_-^a \,, \tag{3.30}$$

with the static gauge $A_\tau = 0$, and look for a solution with $f(-\infty) = 1$ and $f(\infty) = 0$. Equations with $-$ sign reduces to a single ordinary differential equation,

$$\frac{d}{d\tau} f = -\frac{\mu}{3}\left(f - f^2\right), \tag{3.31}$$

and the unique solution satisfying the boundary condition is,

$$f(\tau) = \frac{1}{1+\exp(\mu(\tau - s)/3)} \tag{3.32}$$

where we kept a collective coordinate s explicitly to parameterize where the instanton is centered along the euclidean time.

The value of the euclidean action of this interpolating instanton is

$$S_E = \frac{1}{6}\left(\frac{\mu}{3}\right)^3 \mathrm{Tr}\, J_-^a J_-^a \,. \tag{3.33}$$

Writing J_- as sum of irreducible $n_p \times n_p$ representations with $\sum_p n_p = N$, value of the euclidean action is

$$S_E = \frac{1}{6}\left(\frac{\mu}{3}\right)^3 \sum_p \frac{n_p(n_p-1)^2}{4} \,. \tag{3.34}$$

Such an instanton will contribute to some physical processes but is suppressed by exponentially small factor

$$e^{-S_E} \,. \tag{3.35}$$

Finally, one should remember g_{YM}^2 to recover the actual *dimensionless* exponent is S_E/g_{YM}^2.

4. Zero modes

4.1 Bosonic zero modes

One particular collective coordinate is already manifest in the solution above, which the Goldstone mode coming from time translation symmetry. For any finite action solution to the BPS equation X^a, the translational zero mode,

$$D_\tau X^a \,, \tag{4.1}$$

is a normalizable zero mode. A natural followup question is whether instanton solutions have more bosonic zero modes and whether it is possible to break up an instanton to two or more intermediate ones. For this let us perturb the BPS equation. We may assume static gauge again, $A_\tau = 0$, without loss of generality. Denoting by V^a the perturbed part of X^a, we have

$$\pm\frac{d}{d\tau}V^a = \frac{\mu}{3}V^a - i\epsilon^{abc}[X^b, V^c] \tag{4.2}$$

and the problem reduces to classifying normalizable solutions to this.

The instanton solutions of previous subsection admit relatively simple solutions to this zero mode counting problem. With the right choice of sign, the equation is

$$-\frac{d}{d\tau}V^a = \frac{\mu}{3}\left(V^a + f(\tau)i\epsilon^{abc}[J_-^b, V^c]\right). \tag{4.3}$$

In the far future, $f \to 0$, and the simple exponential behavior due to the $\mu/3$ piece is consistent with the normalizability requirement on zero modes. The main issue is then what happens at past infinity where $f \to 1$.

Two technical points are helpful here. First, one may regard $i\epsilon^{abc}$ as the generators $T_{ac}^{(b)}$ of SU(2). Second, the commutator action with J^a in a representation R of SU(2) actually form a representation $R\otimes R^* \simeq R\otimes R$. Thus the matrix operator that must be diagonalized may be thought of as a triple tensor product

$$K \equiv \sum_{b=5,6,7} T^b \otimes \left(J_-^b \otimes 1 + 1 \otimes J_-^b\right). \tag{4.4}$$

Let us simplify further and consider the case of J_- in the maximal (i.e., spin $(N-1)/2$) irreducible representation. Square of the N dimensional irreducible representation $[(N-1)/2]$ is decomposed as

$$[(N-1)/2]\otimes[(N-1)/2] = [0]\oplus[1]\oplus\cdots\oplus[N-2]\oplus[N-1], \tag{4.5}$$

which gives integer spins only. Tensoring each block $[j]$ with $j = 0, 1, \ldots, N-1$ with the adjoint representation [1] of $T_{ac}^{(b)} = i\epsilon_{abc}$, we find

$$[1]\otimes[j] = [j-1]\oplus[j]\oplus[j+1]. \tag{4.6}$$

Eigenvalues of the operator K for the three blocks on the right hand side are $-(j+1)$, -1, j, respectively. Of these three, only the first can and does produce normalizable zero mode, $2j-1$ number of them, for $j \geq 1$.

Thus, for an instanton that interpolates between a single maximal giant graviton in the irreducible $N\times N$ form and the trivial vacuum, the bosonic zero modes within the same $N\times N$ block may be summarized as in the following table,

degeneracy	1	3	$\cdots$	$2N-5$	$2N-3$
behavior at $-\infty$	$e^{(\mu/3)\tau}$	$e^{2(\mu/3)\tau}$	$\cdots$	$e^{(N-2)(\mu/3)\tau}$	$e^{(N-1)(\mu/3)\tau}$

All of these zero modes decay as $e^{-(\mu/3)\tau}$ at future infinity. There are total $(N-1)^2$ number normalizable bosonic zero modes.

Generally speaking, one might need to take into account of nonnormalizable zero modes in case of euclidean instantons. For solitons, the normalizability requirement arises from consideration of low energy effective action of collective coordinates, since L^2 norm of zero modes appear in the effective kinetic term of the former. For instantons, the only requirement is that we maintain the same finite euclidean action. This by itself does not restrict the zero mode to be normalizable.

In particular, there are N^2-1 number of zero modes from the above (corresponding to $K=-1$) that asymptote to constants at past infinity. Under such a deformation, the instanton solution remain of the same finite action. These zero modes will deform J_- by constant amount, but since small change that respect the vacuum condition cannot alter the representation of SU(2), this deformation cannot change the action. Combined with the above normalizable zero modes, we have total $2N(N-1)$ number of possible deformations of a maximal instanton that preserve the euclidean action. On the other hand, none of the N^2-1 nonnormalizable zero modes are physical. They correspond to rotation of the instanton by global SU(N) unitary transformations. That is, these N^2-1 modes are generated by

$$X^a(\tau) \to gX^a(\tau)g^\dagger\,. \tag{4.7}$$

In the temporal gauge, $A_{10}=0$, we adopted, this is a pure gauge transformation when g is uniform. As a metter of definition of the theory, such a gauge parameter should not be included in the path integral. With more general instantons with both $J_\pm$ nontrivial, there could be relevant nonnormalizable zero modes that rotate one of two $J_\pm$ while leaving the other invariant.

4.2 Fermionic zero modes

Large number of bosonic zero modes hint at similarly large number of fermionic zero modes. Here, we will perform an explicit construction of fermionic zero modes in much the same way as in bosonic case above. The fact that there are 8 broken supersymmetry implies existence of fermionic zero modes arising as Goldstone fermionic modes. These should be of the form

$$\psi_0 = i\left(D_\tau X_a\gamma_a \pm \left(\frac{\mu}{3}X_a - i\epsilon^{abc}X^bX^c\right)\gamma_a\right)\epsilon' = 2i\left(D_\tau X_a\gamma_a\right)\epsilon'\,, \tag{4.8}$$

with wrong chirality fermions ϵ';

$$i\gamma_{567}\epsilon'_0 = \mp\epsilon'_0\,. \tag{4.9}$$

Below we will also recover these zero modes from general fluctuation analysis. Because these arise as Goldstone modes, the associated Grassmann coordinate are special among fermionic ones. Specifically, they cannot have an "interaction vertex" in the effective action of the instanton.

The equation of motion for fermions in the instanton background is

$$D_\tau\Psi + \frac{\mu}{4}(i\gamma_{567})\Psi + \gamma_a[X^a,\Psi] = 0\,. \tag{4.10}$$

Let us again specialize to those instantons which interpolate from giant graviton with representation J_-^a to the trivial vacuum. Normalizability at $\tau=\infty$ requires Ψ to be of positive chirality under $i\gamma_{567}$. Normalizability at $\tau = -\infty$ depends on the last piece, which on the instanton becomes

$$-\frac{\mu}{3} f(\tau)\gamma^a [J_-^a, \Psi]\,, \tag{4.11}$$

so the problem reduces to an eigenvalue problem under the operator

$$\tilde{K} \equiv -\gamma^a \otimes (J_-^a \oplus 1 + 1 \oplus J_-^a)\,. \tag{4.12}$$

A zero mode is normalizable if and only if the eigenvalue of this operator is negative and $< -3/4$.

Because only 3 out of 9 possible γ's are involved, this can also be solved using SU(2) algebra. Let us write the three gamma matrices in a complex form

$$\gamma^a = \sigma^a \otimes \Gamma\,, \tag{4.13}$$

where Γ is some chirality operator such that $\Gamma^2 = 1$. We will use the fact that $i\gamma_{567} = -1 \otimes \Gamma$, meaning that the eigenvalue of Γ is negative of that of $i\gamma_{567}$. Since normalizable zero modes must have positive chirality under the latter, we have in effect,

$$\tilde{K} \equiv \sigma^a \otimes (J_-^a \oplus 1 + 1 \oplus J_-^a)\,, \tag{4.14}$$

where we should remember the extra degeneracy coming from extra spinor indices acted on by Γ. After taking into account that $\Gamma = -1$, this degeneracy gives a factor of 4 when counting real fermionic collective coordinates.

As in bosonic case, we consider the maximal irreducible instanton of size $N \times N$ for J_- side initially. Square of the N-dimensional representations is

$$\left[\frac{(N-1)}{2}\right] \otimes \left[\frac{(N-1)}{2}\right] = [0] \oplus [1] \oplus \cdots \oplus [N-2] \oplus [N-1]\,. \tag{4.15}$$

Tensoring each block $[j]$ with $j = 0, 1, \ldots, N-1$ with a spin half representation $[1/2]$ of $\sigma/2$, we find

$$\left[\frac{1}{2}\right] \otimes [j] = \left[j - \frac{1}{2}\right] \oplus \left[j + \frac{1}{2}\right] \tag{4.16}$$

eigenvalues of the operator $\tilde{K}$ for the three blocks on the right hand side are $-(j+1)$ and j, respectively. Of these two, only the first can and does produce normalizable zero modes, $2j$ number of them, for $j \geq 1$. Thus, for an instanton that interpolates between a single giant graviton in an irreducible $N \times N$ block and the trivial vacuum, the fermionic zero modes may be summarized as in the following table,

degeneracy	4×2	4×4	$\cdots$	$4 \times (2N-4)$	$4 \times (2N-2)$
behavior at $-\infty$	$e^{(5\mu/12)\tau}$	$e^{(3\mu/4)\tau}$	$\cdots$	$e^{(4N-7)(\mu/12)\tau}$	$e^{(4N-3)(\mu/12)\tau}$

All of these zero modes decay as $e^{-(\mu/4)\tau}$ at future infinity. Thus, given an irreducible N-dimensional J_-, The total number of real fermionic collective coordinates from these zero modes within the same $N \times N$ block is $4N(N-1)$.[3]

[3] An unusual fact is that this number of fermionic zero modes match up with the bosonic side, only if we include all $N^2 - 1$ nonnormalizable gauge zero modes in the latter. The latter gives $(N-1)^2 + (N^2-1) = 2N(N-1)$, so one real bosonic mode correponds to one complex fermionic mode. The reason behind this must be that supersymmetry parameter itself has a exponential dependence on τ, although it is not clear to us whether there is a precise one-to-one matching between the two sets.

Of these the first 8 modes are associated with 8 broken supersymmetry. The Goldstone modes of the latter are proportional to

$$iD_\tau X^a \gamma^a e^{(\mu/12)\tau} \epsilon_0' \tag{4.17}$$

with $(i\gamma_{567})\epsilon_0' = \epsilon_0'$. Comparing the asymptotic form of this to the first 8 zero modes, we can see easily that they are of identical form.

4.3 A brief comment on stability of vacua

These 8 fermionic zero modes associated with the broken supersymmetry are quite prohibitive in that instantons amplitudes get further suppression due to them. Generally speaking instanton will contribute to an operator $\mathcal{O}$ via a path integral,

$$\int [dX][d\Psi]\, \mathcal{O}\, e^{-S}. \tag{4.18}$$

Essential part of such a computation comes from integration over bosonic and fermionic collective coordinates, call them z^i and ζ^l, whereby the path integral includes a piece

$$\int \prod_i dz^i \prod_l d\zeta^l\, \mathcal{O}\, e^{-S_E - \delta S(z^i,\zeta^l)} \tag{4.19}$$

Integral over Grassmann numbers ζ^l vanishes unless the integrand produces matching fermions in one-to-on fashion.

For most fermionic zero modes, this is easy to arrange regardless of $\mathcal{O}$, since the higher order correction to euclidean action, $\delta S(z^i, \zeta^l)$, will generally have a potential term involving four (or more) ζ^l's. This does not contradict the statement that ζ^l are zero modes, since the latter simply implies absence of quadratic terms in case of fermions. Such quartic potential term for fermionic collective coordinates has been derived and used extensively in the context of Yang-Mills theory.[4]

Exception to this are Goldstone zero modes, bosonic or fermionic, representing spontaneously broken global symmetries. Nonvanishing amplitude is possible only if one starts with $\mathcal{O}$ with enough fermions in it to cancel Goldstone fermionic coordinates. It shows that there is no mixing among classical vacua by these instantons, and that each classical vacuum of giant gravitons must remain robust against quantum tunneling. Effect of an instanton will show up only at higher order operators.

This, together with stability argument of ref. [14], implies that the classical vacua of giant gravitons are in fact exact quantum mechanical vacua. One could argue that being protected states is by itself enough to guarantee that classical supersymmetric vacua carry over to quantum mechanics, yet, this sort of argument is known to fail in general supersymmetric theories. Such stability argument applies within full quantum theory which is in principle distinct from classical theory no matter how close one gets near the latter.

[4] Direct derivation of such higher order terms with fermionic collective coordinate is particularly well-documented for the case of low energy dynamics of monopoles in 4 dimensions. See ref. [13] for detailed description of how such terms arise.

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There are known cases where, despite classical BPS state, no quantum counterpart exist no matter how close to classical limit one takes [15]. The present theory seems to escape such a possibility, likely due to relatively large number of supersymmetries.

5. General moduli spaces

So far, we studied a rather restricted class of instantons (with $J_+ = 0$), and counted zero modes of even more special case of them, namely those with J_- in the irreducible N-dimensional representation filling out the entire $\mathrm{U}(N)$. We found $(N-1)^2$ normalizable bosonic zero modes, $N^2 - 1$ nonnormalizable one which are unphysical due to gauge symmetry, and $4N(N-1)$ (real) fermion zero modes. As with any quantum theory, one must understand entire species and moduli spaces of instantons to make proper usage of it in path integral, so it is of prime importance to catalog general instantons and their moduli spaces.

For instance, we should ask how the counting changes if we instead consider $n \times n$ irreducible instanton embedded in $\mathrm{U}(N)$ theories. Bosonic zero mode arising from the same $n \times n$ block should follow from the above zero mode counting, so we have $(n-1)^2$ physical modes and $n^2 - 1$ unphysical gauge modes. Repeating the anaylsis of the previous section for "off-diagonal" part of matrices X^a's, we find additional modes. Of these, $2(N-n)(n-2)$ are normalizable and have to be physical. In addition, $2(N-n)n$ modes that approach constant at $\tau = -\infty$ also exist but these are all accounted by global gauge rotation that takes the $n \times n$ solution out into entire $N \times N$. We could repeat this exercise for more general case with more than one irreducible blocks embedded inside $\mathrm{U}(N)$; it is again matter of studying eigenmodes of the algebraic operator K, which become effectively

$$K_{ij} \equiv \sum_{b=5,6,7} T^b \otimes \left(J_i^b \otimes 1 + 1 \otimes J_j^b \right) , \tag{5.1}$$

on different blocks in $\mathrm{U}(N)$ when J_i's are independent irreducible representations of $\mathrm{SU}(2)$ inside J_-.

For classification of instantons and their moduli space, it would be desirable to approach the problem with more global viewpoint. It turns out that this issue was studied in depth previously. An essentially same system of equations was considered before in a different setting of $\mathcal{N} = 1^*$ $\mathrm{U}(N)$ Yang-Mills theory [16]. It just so happens that the vacuum condition and BPS equation for domain walls in $\mathcal{N} = 1*$ theories are identical to our vacuum condition and BPS equation for instantons. They further mapped this system of equations into a mathematical problem of an infinite dimensional hyperKähler quotient procedure and constructed the solution space of the domain walls.

Among the information one could extract from their general approach would be which domain wall (instanton) may split into which combination of domain walls (instantons). On the other hand, counting of deformation was such that some pure gauge mode are counted along with physical ones.[5] Indeed numbers in ref. [16] match precisely with ours whenever

[5]Notably, this is the case when $J_+ = 0$ [16].

the comparison is possible, once we include the global gauge modes in the counting. The constructive approach we took is more detailed in that it gives a simple distinction between the two classes of deformations. Thus, when carefully combined, the two approaches should generate essentially all information of the physical moduli space of instantons, which would be vital if we were computing instanton effect via the Eulidean path integral. In this note, we will not attempt this general anaysis.

6. Vacua and instantons in the presence of fivebranes

A mass-deformation of $\mathcal{N} = 8$ Matrix model including hypermultiplets was derived in [11]. It was constructed to describe longitudinal fivebranes in the pp-wave background. It was also found that there exist many continuous family of classical vacuum solutions which preserve full dynamical symmetries. Here, we will investigate whether there are instanton solutions interpolating between these vacua.

To set up the notation, let us briefly recall what new fields are involved in this $\mathcal{N} = 8$ quantum mechanics. When we have k longitudinal fivebranes probed by $\mathrm{U}(N)$ M(atrix) theory, the only modification to the $\mathrm{U}(N)$ quantum mechanics is introduction of k hypermultiplets in the fundamental representations. This necessarily breaks SUSY down to $\mathcal{N} = 8$ from $\mathcal{N} = 16$, and thus split the adjoint Yang-Mills fields also into two $\mathcal{N} = 8$ multiplets. In the BMN deformed M(atrix) theory considered above, 9 adjoint scalars are already split into 3+6. We labeled the first 3 as $X^{5,6,7}$. In this notation, $X^{6,7,8,9}$ fall under the adjoint hypermultiplet. We further combine these 4 scalars in a complex form,

$$y_2 = X^6 - iX^7 \,, \qquad y_1 = X^8 - iX^9 \,. \tag{6.1}$$

Whenever we introduce an longitudinal fivebrane sitting at the center, we should add a fundamental hypermultiplet with two complex scalars,

$$q_2^{(f)} \,, \qquad q_1^{(f)} \,. \tag{6.2}$$

Our convention is such that q's are anti-fundamental representations while their conjugates, $\bar{q}$ and $\bar{q}$, are in the fundamental representation.

We should note here that ref. [11] found two distinct class of vacuum solutions invariant under all supersymmetries. One set involving X^5, y_2, q_2 give obvious generalization of the Giant graviton in the bulk, while the other involves X^5, y_2, q_1. In the following we found instanton solutions associated with the former only. There appears to be no supersymmetric instanton solution associated with the latter.

6.1 Abelian case

For simplicity, we first consider the instanton solution interpolating between an abelian giant graviton at past infinity and trivial vacuum at future infinity. Turning off irrelevant

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fields, we write down the euclidean action in a complete square form,

$$
\begin{aligned}
S_E &= \frac{1}{2}\int d\tau \mathrm{Tr}\left\{\left(\mp D_\tau X_5 + \left(\frac{1}{2}\sum_f \bar{q}_2^{(f)} q_2^{(f)} + \frac{\mu}{3}X_5\right)\right)^2 + \right.\\
&\qquad \left. + \sum_f \left(\mp D_\tau \bar{q}_2^{(f)} + \left(X_5 + \frac{\mu}{3}\right)\bar{q}_2^{(f)}\right)\left(\mp D_\tau q_2^{(f)} + q_2^{(f)}\left(X_5 + \frac{\mu}{3}\right)\right)\right\} \pm \\
&\quad \pm\frac{1}{2}\int d\tau \mathrm{Tr}\left\{(2D_\tau X_5)\left(\frac{1}{2}\sum_f \bar{q}_2^{(f)} q_2^{(f)} + + \frac{\mu}{3}X_5\right) + \right.\\
&\qquad \left. + \sum_{(f)} D_\tau \bar{q}_2^{(f)}\left(X_5 + \frac{\mu}{3}\right) q_2^{(f)} + \sum_f \left(X_5 + \frac{\mu}{3}\right)\bar{q}_2^{(f)} D_\tau q_2^{(f)}\right\}\\
&\geq \pm\frac{1}{2}\int d\tau \frac{d}{d\tau}\mathrm{Tr}\left(\frac{\mu}{3}X_5^2 + \frac{\mu}{3}\sum_f \bar{q}_2^{(f)} q_2^{(f)} + X_5 \sum_f \bar{q}_2^{(f)} q_2^{(f)}\right)\\
&= \pm\frac{1}{2}\mathrm{Tr}\left(\frac{\mu}{3}X_5^2 + \frac{\mu}{3}\sum_f \bar{q}_2^{(f)} q_2^{(f)} + X_5 \sum_f \bar{q}_2^{(f)} q_2^{(f)}\right)\Bigg|_{-\infty}^{+\infty} .
\end{aligned}
\tag{6.3}
$$

Lower bound is saturated if the first order equations are satisfied,

$$
\begin{aligned}
\pm D_\tau X_5 &= \frac{1}{2}\sum_f \bar{q}_2^{(f)} q_2^{(f)} + \frac{\mu}{3}X_5 ,\\
\pm D_\tau \bar{q}_2^{(f)} &= X_5 \bar{q}_2^{(f)} + \frac{\mu}{3}\bar{q}_2^{(f)} .
\end{aligned}
\tag{6.4}
$$

Dropping time dependence, we recover a Giant graviton of the form

$$
X_5 = -\frac{\mu}{3}, \qquad q_2^{(f)} = \sqrt{2}\frac{\mu}{3} z^f , \tag{6.5}
$$

where z is any complex k-vector of unit length. Vacuum moduli space is CP^{k-1}.

The ansatz for the instanton solution interpolating between an abelian giant graviton at past infinity and perturbative vacuum at future infinity is

$$
X_5(\tau) = -\frac{\mu}{3}f(\tau), \qquad \bar{q}_2^{(f)}(\tau) = \sqrt{2}\frac{\mu}{3} z^f g(\tau), \tag{6.6}
$$

with $f(-\infty) = 1 = g(-\infty)$ and $f(+\infty) = 0 = g(+\infty)$. Inserting it into the first order equations with $-$ sign, we get

$$
\dot{f} = -\frac{\mu}{3}\left(f - g^2\right), \qquad \dot{g} = -\frac{\mu}{3}\left(g - fg\right). \tag{6.7}
$$

A unique solution to this with the right asymptotic behavior is (3.31)

$$
f = g = \frac{1}{1 + \exp(\mu(\tau - s)/3)}, \tag{6.8}
$$

whose form is identical to the above $\mathcal{N} = 16$ case. The action is

$$
S_E = \frac{1}{2}\left(\frac{\mu}{3}\right)^3 . \tag{6.9}
$$

This solution has a single bosonic zero mode which corresponds to a Goldstone mode.

6.2 General case

We now repeat the exercise for non-abelian case and consider instanton solutions interpolating between a nonabelian giant graviton at past infinity and trivial vacuum at future infinity. Turning on adjoint scalars X_5, $\bar{y}_2$ and fundamental scalars $\bar{q}_2^{(f)}$ only, we have the following inequality of euclidean action,

$$S_E \geq \pm\frac{1}{2}\mathrm{Tr}\left(\frac{\mu}{3}X_5^2 + \frac{\mu}{3}\sum_f \bar{q}_2^{(f)}q_2^{(f)} + \frac{\mu}{3}\bar{y}_2 y_2 + X_5\sum_f \bar{q}_2^{(f)}q_2^{(f)} + X_5\left[\bar{y}_2, y_2\right]\right)\Bigg|_{-\infty}^{+\infty}. \tag{6.10}$$

The bound is saturated if and only if

$$\begin{aligned}
\pm D_\tau X_5 &= \frac{1}{2}\sum_f \bar{q}_2^{(f)}q_2^{(f)} + \frac{\mu}{3}X_5 + \frac{1}{2}\left[\bar{y}_2, y_2\right], && (6.11)\\
\pm D_\tau \bar{q}_2^{(f)} &= X_5\bar{q}_2^{(f)} + \frac{\mu}{3}\bar{q}_2^{(f)}, && (6.12)\\
\pm D_\tau \bar{y}_2 &= [X_5, \bar{y}_2] + \frac{\mu}{3}\bar{y}_2\,. && (6.13)
\end{aligned}$$

For the instanton solution interpolating between an nonabelian giant graviton occupying an $n \times n$ block of adjoint scalars at past infinity and perturbative vacuum at future infinity, we use an ansatz of the form,

$$\begin{aligned}
-(X_5)_{AB}(\tau) &= \frac{\mu}{3}(m+1-A)f(\tau)\delta_{AB}\,,\\
(\bar{q}_2^{(f)})_A(\tau) &= \sqrt{2}\frac{\mu}{3}\omega_2^f g(\tau)\delta_{A,m}\,,\\
(\bar{y}_2)_{AB}(\tau) &= \sqrt{2}\frac{\mu}{3}\alpha_A h(\tau)\delta_{A,B-1}\,, && (6.14)
\end{aligned}$$

where

$$|\alpha_A|^2 - |\alpha_{A-1}|^2 = m+1-A\,, \tag{6.15}$$

for all $1 \leq A \leq n$ except for $A = m$

$$\sum |w_2^f|^2 + |\alpha_m|^2 - |\alpha_{m-1}|^2 = 1\,, \tag{6.16}$$

Here m is any fixed integer such that $(n-1)/2 < m \leq n$.

To find an instanton that interpolate between a giant graviton and perturbative vacua, we set the boundary conditions,

$$\begin{aligned}
f(-\infty) = g(-\infty) = h(-\infty) &= 1\,,\\
f(+\infty) = g(+\infty) = h(+\infty) &= 0\,. && (6.17)
\end{aligned}$$

Taking an appropriate sign choice, and writing equation for X^5 component-wise, we find

$$\dot{f} = -\frac{\mu}{3}\left(f - h^2\right), \tag{6.18}$$

from all diagonal entries except for $A = m$ case. The latter gives a slightly different equation

$$\dot{f} = -\frac{\mu}{3}\left(f - h^2\right) + \frac{\mu}{3}|\omega_2|^2\left(g^2 - h^2\right). \tag{6.19}$$

Two equations are consistent only if $g^2 = h^2$, which in turn forces $g = h$. Then equations for y_2 and for q_2 collapse to one,

$$\dot{h} = -\frac{\mu}{3}\left(h - fh\right). \tag{6.20}$$

Unique solution consistent with the boundary condition is

$$f = g = h = \frac{1}{1 + \exp(\mu(\tau - s)/3)}. \tag{6.21}$$

Again, dependence on τ is identical to $\mathcal{N} = 16$ case. Value of the euclidean action also has a simple expression,

$$S_E \geq -\frac{1}{2}\left(\frac{\mu}{3}\right)\mathrm{Tr}(X_5)^2\Big|_{-\infty}^{+\infty}. \tag{6.22}$$

This is essentially of the same form as the euclidean action in $\mathcal{N} = 16$ theory. With large-N, it again scales as $\sim (\mu n)^3$.

6.3 Supersymmetry

As with $\mathcal{N} = 16$ case, instantons of $\mathcal{N} = 8$ theories also preserves some supersymmetry. Euclideanization of the theory and the supersymmetry proceed similarly, and we have

$$\begin{aligned}
\delta\lambda_1 &= iD_\tau X_\mu\gamma_\mu\epsilon_1 - i\sum_{\mu,\nu}^{5}\frac{1}{2}[X_\mu, X_\nu]\gamma_{\mu\nu}\epsilon_1 + iD^3\epsilon_1 + (iD^1 + D^2)\epsilon_2 - \\
&\quad -i\frac{\mu}{6}\sum_{\mu=1}^{4}X_\mu\gamma_\mu\gamma_5\epsilon_1 + i\frac{\mu}{3}X_5\epsilon_1\,, \\
\delta\chi_1 &= -iD_\tau y_i\epsilon_i - i\sum_{\mu=1}^{5}[X_\mu, y_i]\gamma_\mu\epsilon_i + \frac{i\mu}{6}\gamma_5 y_1\epsilon_1 + \frac{i\mu}{3}\gamma_5 y_2\epsilon_2\,, \\
\delta\psi_1 &= -iD_\tau q_i\epsilon_i + i\sum_{\mu=1}^{5}q_i X_\mu\gamma_\mu\epsilon_i + \frac{i\mu}{6}\gamma_5 q_1\epsilon_1 + \frac{i\mu}{3}\gamma_5 q_2\epsilon_2\,,
\end{aligned} \tag{6.23}$$

and similar expressions for $\delta\lambda_2$, $\delta\chi_2$, and $\delta\psi_2$. $D^{1,2,3}$ are familiar auxiliary fields of vector multiplet. λ_i, χ_i, and ψ_i are 4-component Dirac spinors from vector multiplet, adjoint hypermultiplet, and fundamental hypermultiplets.[6] To recover $\delta\lambda_2$, $\delta\chi_2$, and $\delta\psi_2$, one

[6]In Minkowski signature, they are each symplectic Majorana such that

$$\lambda_1 = -i\tilde{B}\lambda_2^*\,, \qquad \chi_1 = +i\tilde{B}\chi_2^*\,, \qquad \psi_1 = +i\tilde{B}\psi_2^* \tag{6.24}$$

with some conjugation matrix $\tilde{B}$. Similarly, supersymmetry parameter ϵ_i are symplectic Majorana spinors,

$$\epsilon_1 = -i\tilde{B}\epsilon_2^*\,, \tag{6.25}$$

Once euclideanized, however, this Majorana condition cannot be imposed and we must treat two components under i index independently. As before, correct degrees of freedom is found by giving up hermiticity and treating the path integral as if it is a contour integral.

must care to euclideanize *after* performing charge conjugation. For the instantons in question, this reduces to

$$\begin{aligned}
\delta\lambda_1 &= +iD_\tau X_5\gamma_5\epsilon_1 + iD^3\epsilon_1 + i\frac{\mu}{3}X_5\epsilon_1\,,\\
\delta\chi_1 &= -iD_\tau y_2\epsilon_2 - i[X_5,y_2]\gamma_5\epsilon_2 + \frac{i\mu}{3}\gamma_5 y_2\epsilon_2\,,\\
\delta\psi_1 &= -iD_\tau q_2\epsilon_2 + iq_2X_5\gamma_5\epsilon_2 + \frac{i\mu}{3}\gamma_5 q_2\epsilon_2\,. \qquad (6.26)
\end{aligned}$$

On shell, the value of D^3 is

$$D^3 = \frac{1}{2}\sum_f \bar{q}_2^{(f)}q_2^{(f)} + \frac{1}{2}\left[\bar{y}_2, y_2\right]. \qquad (6.27)$$

Instanton solutions then preserves half of supersymmetry. Preserved supersymmetry are generated by ϵ_1^+ and ϵ_2^- where

$$\gamma_5\epsilon_i^\pm = \pm\epsilon_i^\pm\,, \qquad (6.28)$$

which also imply existence of 4 Goldstone fermionic zero modes generated by the other half ϵ_1^- and ϵ_2^+. This gives rise to 4 real Grassmannian collective coordinates, which is free of any interaction vertex. While larger instanton solutions will admit more zero modes, bosonic and fermionic, this 4 plus a single bosonic coordinate from translational symmetry will remain special in that they remain free no matter what.

7. Instantons in $AdS \times S$ and penrose limit

PP-wave background in eleven dimension is obtained from $AdS_7 \times S^4$ or $AdS_4 \times S^7$ by taking the Penrose limit. Instantons interpolating between giant graviton solutions in $AdS \times S$ spaces were found in [17]. We take Penrose limit of $AdS \times S$ space and compare the limiting form of instantons to those in the BMN M(atrix) theory.

First, We briefly summarize Penrose limit of $AdS_7 \times S^4$ and $AdS_4 \times S^7$. Global metric for $AdS_7 \times S^4$ is

$$\begin{aligned}
F_4 &= 3R_S^3\mathrm{vol}(S^4)\\
ds^2 &= R_{AdS}^2\left(-\cosh^2\rho d\tau^2 + d\rho^2 + \sinh^2\rho d\Omega_5^2\right) + R_S^2 dS_4^2\,, \qquad (7.1)
\end{aligned}$$

where

$$dS_n^2 = d\theta_n^2 + \sin^2\theta_n^2 dS_{n-1}^2\,. \qquad (7.2)$$

Here $R_{AdS} = 2R_S = 2l_p(\pi N)^{\frac{1}{3}}$ and l_p is eleven dimensional Planck length. Penrose limit is taken with $R_S \to \infty$ (or $N \to \infty$) following the reparametrizations

$$\begin{aligned}
\theta_i &= \frac{\pi}{2} - \frac{y_i}{R_S} \quad (i = 1,2,3)\,,\\
\rho &= \frac{z}{R_{AdS}}\,,\\
\tau &= \frac{\mu x^+}{6} + \frac{6x^-}{R_{AdS}^2\mu}\,,\\
\theta_4 &= \frac{\mu x^+}{3} - \frac{3x^-}{R_S^2\mu}. \qquad (7.3)
\end{aligned}$$

Finally we get the plane wave geometry

$$
\begin{aligned}
ds^2 &= -4dx^+dx^- - \left(\left(\frac{\mu}{3}\right)^2 y^2 + \left(\frac{\mu}{6}\right)^2 z^2\right) dx^{+2} + d\vec{y}^2 + d\vec{z}^2 \,, \\
F_4 &= \mu dx^+ \wedge dy^1 \wedge dy^2 \wedge dy^3 \,.
\end{aligned} \tag{7.4}
$$

Global metric for $AdS_4 \times S^7$ is

$$
\begin{aligned}
ds^2 &= R_{AdS}^2 \left(-\cosh^2\rho d\tau^2 + d\rho^2 + \sinh^2\rho d\Omega_2^2\right) + R_S^2 dS_7^2 \,, \\
F_4 &= 3vol(AdS_4) \,.
\end{aligned} \tag{7.5}
$$

Here $R_{AdS} = 1/2R_S = 1/2l_p(32\pi^2 N)^{\frac{1}{6}}$. Again Penrose limit is taken with the limit $R_S \to \infty$ after the reparametrizations

$$
\begin{aligned}
\theta_i &= \frac{\pi}{2} - \frac{z_i}{R_S} \quad (i = 1, \cdots, 6) \,, \\
\rho &= \frac{y}{R_{AdS}} \,, \\
\tau &= \frac{\mu x^+}{3} + \frac{3x^-}{R_{AdS}^2 \mu} \,, \\
\theta_7 &= \frac{\mu x^+}{6} - \frac{6x^-}{R_S^2 \mu} \,.
\end{aligned} \tag{7.6}
$$

We get the same geometry as (7.4).

Instanton solutions interpolating between perturbative vacuum and spherical membranes in pp-wave background have two-fold origins in $AdS \times S$ space: (1) One coming from instantons interpolating between perturbative vacuum and rotating spherical brane in AdS_4 of $AdS_4 \times S^7$ and (2) the other coming from instantons interpolating between perturbative vacuum and rotating spherical brane in S^4 of $AdS_7 \times S^4$. The form of solutions are presented in [17]. Taking Penrose limit of case (2), we find that the limiting form of instanton is identical to that obtained in Matrix theory:

$$
y = \frac{R_s p}{1 + \exp(\mu\tau/3)} \,, \tag{7.7}
$$

where p is Penrose limit of angular momentum defined in [17]. Instanton interpolates between $y_0 = 0$ and $y_p = R_s p$. Value of the euclidean action is

$$
S_E \approx \frac{1}{6} N p^3 = \frac{R_s^3 p^3}{6\pi l_p^3} = \frac{y_p^3}{6\pi l_p^3} \,. \tag{7.8}
$$

For the case of (1), the Penrose limit of the instanton asymptotes to

$$
y \sim \frac{R_{AdS}\, \tilde{p}}{1 + \exp(\mu\tau/3)} \,. \tag{7.9}
$$

$\tilde{p}$ is again angular momentum of giant graviton in the AdS space. Value of the euclidean action,

$$
S_E \approx \frac{\sqrt{2}}{12} \sqrt{N} \tilde{p}^3 = \frac{R_s^3}{48\pi l_p^3} \tilde{p}^3 = \frac{y_{\tilde{p}}^3}{6\pi l_p^3} \,, \tag{7.10}
$$

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so we have the same value of the action both cases. This is consistent with the fact that we have the same pp-wave limits for both $AdS_7 \times S^4$ and $AdS_4 \times S^7$.

As an aside, we may consider instanton solutions which interpolate between perturbative vacuum and spherical fivebranes. Again we note that S^5 giant gravitons could arise from two sources: (3) fivebranes in S^7 of $AdS_4 \times S^7$ and (4) fivebranes in AdS_7 of $AdS_7 \times S^4$. For the case (3), the instanton after taking Penrose limit has the form

$$z^4 = \frac{R_s^4 p}{1 + \exp(\mu\tau/6)} \,. \tag{7.11}$$

while the euclidean action is evaluated to be

$$S_E \approx \frac{1}{3} N p^{\frac{3}{2}} = \frac{R_s^6}{96\pi^2 l_p^6} p^{\frac{3}{2}} = \frac{z_p^6}{96\pi^2 l_p^6} \,. \tag{7.12}$$

For the case (4), the solution again takes the same asymptotic form as in case (3). Value of euclidean action is again,

$$S_E \approx \frac{2}{3} N^2 \tilde{p}^{\frac{3}{2}} = \frac{2R_s^6}{3\pi^2 l_p^6} \tilde{p}^{\frac{2}{3}} = \frac{z_{\tilde{p}}^6}{96\pi^2 l_p^6} \,. \tag{7.13}$$

As with S^2 case, we get the same value of the actions for instantons of different origins. Note that $S_E \sim x^3$ for instantons interpolating between membranes and $S_E \sim x^6$ for instantons interpolating between fivebranes. The values of action just count the corresponding euclidean volume of bubble creation and agree with the dimensional analysis. Recently it was proposed [18] that S^5 giant graviton vacua and S^2 giant graviton vacua are dual to each other in this Penrose limit. This would imply that the two classes of instantons we considered above should be related to each other in the Penrose limit. However, the hypothesis is not amenable to the naive interpolation of S^5 giant graviton from $AdS \times S$ to the Penrose limit. This fact is also clear from behavior of the instanton solution above. If there is such a correspondence, it is apparently not visible in a semiclassical approach, as was noted in ref. [18].

8. Summary

In this note, we studied euclideanized BMN M(atrix) theory and constructed instanton solutions that interpolate between classically distinct vacua. We considered two kinds of M(atrix) theory. One is the original BMN M(atrix) theory with $\mathcal{N} = 16$ supersymmetries, while the other with reduced $\mathcal{N} = 8$ supersymmetry corresponds to having longitudinal fivebranes in the background.

We gave equations for supersymmetric instantons, and solved them for all possible instantons connecting to the trivial perturbative vacuum. We gave detailed counting and, in some cases, explicit construction of zero modes around such solutions. This includes Goldstone modes. and due to some of fermionic Goldstone modes, instantons cannot mediate mixing of vacua and each classical vacuum remain robust against quantum tunneling. Instantons will certainly contribute to higher order interaction vertices, nevertheless. It remains an open question as to what specific roles instantons will play in uncovering quantum nature of BMN M(atrix) theory.

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